DN-003-001408

Seat No. ________

Bsc. (Sem. IV) (CBCS) Examination

March-2022

Mathematics : BSMT-401(A)

(Advanced Calculus & Linear Algebra)

(Old Course)

Faculty Code : 003

Subject Code : 001408

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 Answer the following objective type questions briefly in your answer-book. **20**

(1) Define rectangular neighborhood.

(2) If $\lim\limits_{(x,y)\to(a,b)} f(x,y)=l$ and (2) function g is continuous at $x=a$ and $g(a)=b$ then what is the value of $\lim\limits_{(x,y)\to(a,b)} f(x,y)$?

(3) Write domain of $\frac{f}{g}$ where f and g are real valued two variable functions and their domains are D_1 and D_2

(4) Define Continuity of two variable function.

(5) If $f(x,y)=0$, then write $\frac{dy}{dx}$ in terms of partial derivatives.

(6) Write Jacobian of u & v with respect of x and y.

(7) If $f(x)=x\tan\frac{y}{x}$ then find the value of $f_x(4,\pi)$

(8) Define grad $\varnothing$

(9) Find the unit normal at (1, 1, 1) if $\varnothing(x,y,z)=x^2y+y^2x+z^2$.

(10) If the vector $\overline{V}=(ax^2)\hat{l}+(bxy)\hat{j}+(cy^2z-3yz)\hat{k}$ is solenoidal then find its divergence.

(11) Evaluate $\iint_R xy\,dx =$ where $R = \{x, y) \mid x^2 \le y \le \sqrt{x}, 0 \le x \le 1\}$.

(12) Change the order and write : $\int_0^1 \int_{x^2}^1 f(x, y)dydx.$

(13) Evaluate $\int_0^1 \int_0^1 \int_0^1 (x+y+z)dxdydz$.

(14) Evaluate $\int_0^a \int_0^{2\pi} \int_0^{\pi} r^2 \sin\phi \, drd\theta \, d\phi$

(15) Evaluate $\int_0^1 \int_0^1 \frac{dydx}{(1+x^2)(1+y^2)}$

(16) What is the value of $\int_c \frac{dx}{x+y}$, where $C: x = at^2, y = 2at, 0 \le t \le 2.$

(17) Write the general form of a line integral.

(18) What is the value of $\Gamma\left(\frac{1}{2}\right)$.

(19) If $u = (2, -2, 1) \in R^3$ where R^3 is an Euclidean space then $\| u \| =$ __________.

(20) V is an inner product space $\bar{u}, \bar{v} \in$ V and $a \in R$ then $\| a\bar{u} \| =$ __________.

2 (A) Attempt Any **Three** : **06**

(1) Discuss the existance of $\lim\limits_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ by derving iterated limits.

(2) If $x = r\cos\theta, y = r\sin\theta$ then show that J=r.

(3) If $H = f(x, y, z)$ *and* $x = g(u, v, w), y = h(u, v, w)$ *and* $z = \phi\,(u, v, z)$ *then write* $\frac{\partial H}{\partial u}, \frac{\partial H}{\partial v}$ *and* $\frac{\partial H}{\partial w}$

(4) What is Local/Relative Maxima ?

(5) For a scalar function ϕ on D prove that $curl\,(grad\phi) = \bar{0}$.

(6) Find $\nabla\phi$ where $\phi = \log(x^2 + y^2 + z^2)$.

2 (B) Attempt Any **Three** : **09**

(1) If $f(x, y) = xy; x, y \in R,$ then discuss the continuity of at (1, 2)

(2) If $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ then find approximate value of $f(1.9, 2.01, 4.8)$.

(3) If $z = f(x, y), x = e^u + e^{-v}$ *and* $y = e^{-u} + e^u$ *then show that* $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$

(4) If $u = \tan^{-1}\frac{x^3 + y^3}{x - y}; x \neq y$ then find $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{dy^2}$

(5) If $\overline{f} = x^2 y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$ than find $curl\,(curl\,\overline{f})$

(6) If $\overline{f}$ is solenoidal functions then find a where $\overline{f} = (ax + 3y + 4z)\hat{i} + (x - 2y + 3z)\,\hat{j} + (3x + 2y - z)\hat{k}$

2 (C) Attempt any **Two** **10**

(1) State and prove Euler's theorem.

(2) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$

(3) Find the maximum value of $f(x, y, z) = xyz$ subject to the constraint $2x + 2y + z = 108.$

(4) If $f(x, y) = \frac{1}{\sqrt{y}}e^{-\frac{(x-a)^2}{4y}}$ then show that $f_{xy} = f_{yx}$.

(5) Prove that $\nabla^2(\varphi\psi) = \phi\nabla^2\psi + 2\nabla\varphi\psi + \psi\nabla^2\varphi$

3 (A) Attempt any **three** : **06**

(1) Find $d(u, v)$ where $u = (-1, 2)$ and $v = (2, 5)$ using Euclidean inner product.

(2) Evaluate $\oint_c xdy - ydx$ where C is the circle $x^2 + y^2 = 1$.

(3) Find $curl\ f\ if\ f = x^2 y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$.

(4) Prove that $p\beta(p, q+1) = q\beta(p+1, q)$.

(5) Prove that the line integral $\int_{(0,0)}^{(x,y)} (6xy^2 - y^3)dx + (6x^2y - 3xy^2)dy$ is independent of path.

(6) Evaluate $\int_0^1 \int_0^2 (x^2 + 2u)dxdy$

3 (B) Answer in details : (Any **Three**) **09**

(1) Evaluate $\iiint_R \sqrt{x^2 + y^2}\, dxdydz$ where **R** is upper part of the cone $x^2 + y^2 = z^2$ and the plane $z = 1$.

(2) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z dydx$.

(3) Prove that $\int_0^1 \frac{x^5 dx}{1 - x^4} = \frac{\pi}{8}$

(4) Prove that $div(\phi f) = (grad\phi) \bullet \mathbf{f} + \phi div\mathbf{f}$

(5) Obtain the area of ellipse using Green's theorem.

(6) Show that $(u,v) = 9u_1v_1 + 4u_2v_2$ is the inner product on R^2 generated by the matrix $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$.

3 (C) Write a note on : (Any **Two**) **10**

(1) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} xdxdy$

(2) State and prove Green's theorem for a plane.

(3) State and prove relation between Beta and Gamma functions.

(4) Using Stok's theorem find $\iint_s x^3dydz + y^3dzdx + z^3dxdy$, where S is a surface of sphere $x^2 + y^2 + z^2 = a^2$.

(5) Find an orthonormal basis for R^3 containing the vectors $v_1 = (3, 5, 1)$ and $v_2 = (2, -2, 4)$ using the Euclidean inner product.